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PJLZ–gauge fixing approach in $SU(2)$ lattice gauge theory. [†]

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Abstract

We study the $SU(2)$ gauge theory with the interpolating gauge *a la* Parrinello–Jonas–Lasinio–Zwanziger (PJLZ) with the gauge fixing functional $F = \sum_{x\mu} \frac{1}{2} \text{Tr}(U_{x\mu} \sigma_3 U_{x\mu}^\dagger \sigma_3)$. We find a strong indication of the non-analiticity with respect to the interpolating parameter λ at $\lambda_c \sim 0.8$.

1 Introduction

Gauge variant objects, i.e. Green functions for gluons or/and quarks, are among the most popular objects in continuum physics. A comparison of nonperturbatively calculated Green functions on the lattice with continuum (mainly, perturbative) ones can give an insight on the structure of the lattice theories and role of nonperturbative effects. Another important point is that Green functions are supposed to contain information about the physical ‘observables’ which must not depend on the gauge chosen, e.g. dynamical gluon masses, screening masses, etc.. Therefore, it is important to disentangle gauge–dependent features from gauge independent ones.

A somewhat special reason to study the gauge, interpolating between no–gauge and Maximally Abelian gauge (MAG), is connected with a fate of the so called Abelian Dominance. Recently Ogilvie has shown [1] (see also [2]) that for Abelian Projection (AP) the gauge fixing is unnecessary, i.e. AP without gauge fixing yields the exact string tension of the underlying non–Abelian theory : $\sigma_{Abel} = \sigma_{SU(2)}$.

These observations shed a new light on the problem of Abelian Dominance. Indeed, without MAG the Abelian Projection ensures the exact equality between σ_{Abel} and $\sigma_{SU(2)}$ while with MAG Abelian σ_{Abel} and full $\sigma_{SU(2)}$ string tensions are close but not equal : $\sigma_{Abel} \neq \sigma_{SU(2)}$, at least for β –values employed (see, e.g. [3, 4]).

The question arises if it is possible to interpolate ‘smoothly’ from the no–gauge case to the gauge fixed case. To answer this question is the main goal of this work.

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2 Gauge fixing procedure and algorithm

We consider the pure gauge $SU(2)$ theory with standard Wilson action $\beta \cdot S(U)$ [7]. According to P JLZ–approach [5],[6] the average of any gauge–noninvariant functional $\mathcal{O}(U)$ is given by

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [dU] \tilde{\mathcal{O}}(U; \lambda) \cdot e^{-\beta S(U)}, \quad (1)$$

where $\tilde{\mathcal{O}}(U; \lambda) = \langle \mathcal{O} \rangle_\Omega$ and

$$\begin{aligned} \langle \mathcal{O} \rangle_\Omega &= \frac{1}{I(U; \lambda)} \int [d\Omega] \mathcal{O}(U^\Omega) \cdot e^{\lambda F(U^\Omega)}; \\ I(U; \lambda) &= \int [d\Omega] e^{\lambda F(U^\Omega)}, \end{aligned} \quad (2)$$

where $F(U)$ is a gauge fixing functional and $U_{x\mu}^\Omega = \Omega_x U_{x\mu} \Omega_{x+\mu}^\dagger$. We have chosen

$$F = \sum_{x\mu} \frac{1}{2} \text{Tr}(U_{x\mu} \sigma_3 U_{x\mu}^\dagger \sigma_3). \quad (3)$$

In eq.(2) the functional $F_U(\Omega)$ plays the role of effective action with unitary ‘spins’ Ω_x and random bonds described by fields $U_{x\mu}$ (similar to spin–glass model).

Evidently, the maximization of $F(U^\Omega)$ with respect to gauge transformations Ω defines MAG, and for gauge invariant functional $\tilde{\mathcal{O}}(U; \lambda) = \mathcal{O}(U; \lambda)$.

In eq.’s (2) λ is some ‘interpolating’ parameter between 0 and ∞ . The choice $\lambda = 0$ corresponds to the no–gauge case and the limit $\lambda \rightarrow \infty$ corresponds to the case of the Maximally Abelian gauge. Any physical, i.e. gauge invariant, observable (screening masses, etc.) must not depend on λ . In general, there is no grounds for saying that one value of λ is more physical than another value. However, things can be different in the case of the Abelian Projection if $\lambda = 0$ and $\lambda = \infty$ belong to two different phases.

In our study we use $\mathcal{O} = F(U)$ defined in eq.(3) and $F_{norm}(U) = F(U)/4V_4$. In the ‘strong coupling’ approximation ($\lambda \sim 0$) one obtains

$$\langle F_{norm} \rangle_{str} = \lambda/3 + \dots, \quad (4)$$

where V_4 is the number of sites.

Definitions in eq.’s (1)÷(2) presume the following numerical algorithm [8].

- i) Generate a set of link configurations $\{U_{x\mu}^{(1)}\}, \{U_{x\mu}^{(2)}\}, \dots$ using standard gauge invariant algorithm with Wilson action $S(U)$ at some value of β .
- ii) For every configuration $\{U_{x\mu}^{(i)}\}$ generate sequence of configurations $\{\Omega_x^{(1)}\}, \{\Omega_x^{(2)}\}, \dots$ weighted by the factor $\exp(\lambda F(U^\Omega))$ at some value of λ . Therefore, one obtains the estimator for $\tilde{\mathcal{O}}(U; \lambda) = \langle \mathcal{O} \rangle_\Omega$

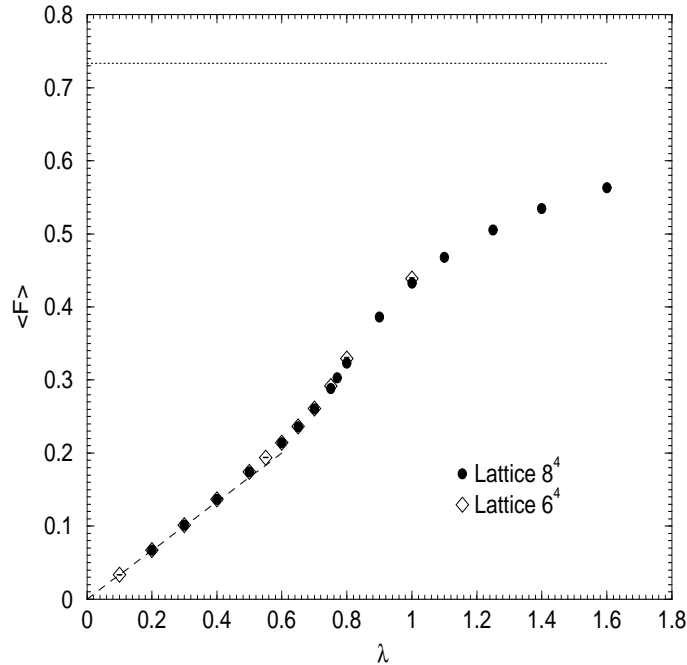


Figure 1: The dependence of the $\langle F_{norm} \rangle$ on λ . Symbols are explained in the text.

$$\tilde{\mathcal{O}}(U; \lambda) = \frac{1}{N_{\Omega}} \sum_{j=1}^{N_{\Omega}} \mathcal{O}(U^{\Omega(j)}) . \quad (5)$$

iii) The estimator for the expectation value $\langle \mathcal{O} \rangle$ is obtained as

$$\langle \mathcal{O} \rangle = \frac{1}{N_U} \sum_{i=1}^{N_U} \tilde{\mathcal{O}}(U^{(i)}; \lambda) . \quad (6)$$

3 Numerical results

The most part of our calculations has been performed on the 8^4 lattice at $\beta = 2.4$. Some calculations have been done also on the 6^4 and 10^4 lattices to control finite volume effects.

In Figure 1 one can see the dependence of the $\langle F_{norm} \rangle$ on λ at $\beta = 2.4$. The dashed line corresponds to the lowest order ‘strong coupling’ approximation $\langle F \rangle_{strong} = \lambda/3$. The upper dotted line corresponds to Maximally Abelian gauge. The agreement between numerical data and ‘strong coupling’ expansion in eq.(4) is very good up to $\lambda \simeq 0.6$. $\langle F_{norm} \rangle$ has a clear change of regime at $\lambda \sim 0.8$. It is interesting to note that this dependence is very similar to that for another choice of the functional $F = F_{LG}$ [8] which corresponds to the Lorentz (or Landau) gauge at infinite values of the interpolating parameter.

For any $\{U_{x\mu}\}$ -configuration the ‘specific heat’ $C(U; \lambda)$ is defined as

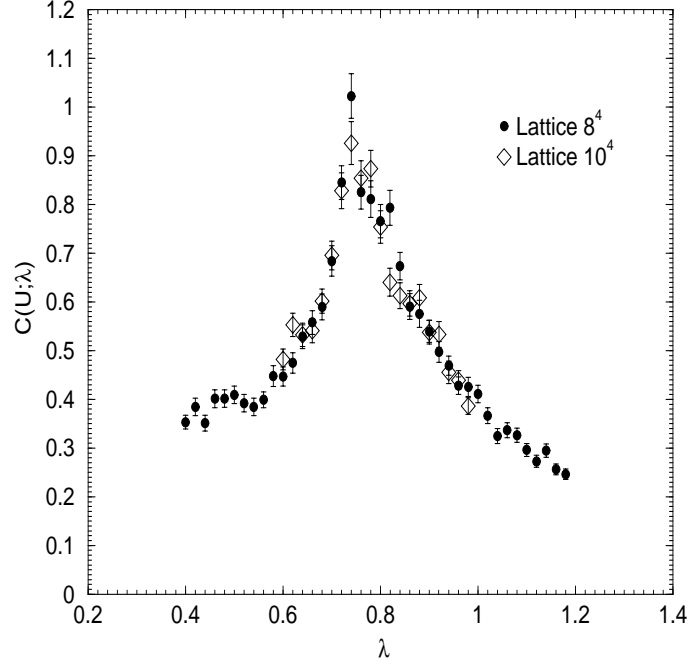


Figure 2: The dependence of $C(U; \lambda)$ on λ for some typical configuration $\{U_{x\mu}\}$.

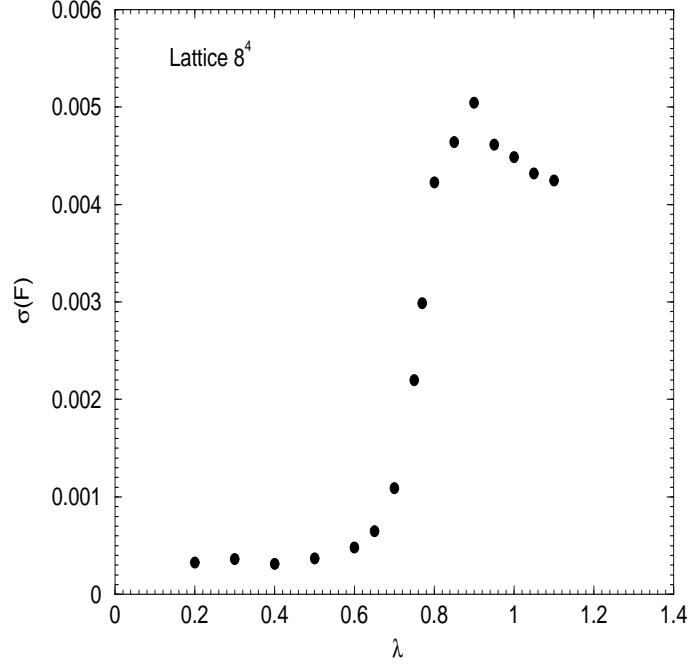


Figure 3: The dependence of $\sigma(F)$ on λ .

$$C_U(\lambda) = \frac{1}{4V_4} \frac{d\tilde{F}(U; \lambda)}{d\lambda} = \frac{\langle F^2 \rangle_\Omega - \langle F \rangle_\Omega^2}{4V_4}. \quad (1)$$

In Figure 2 we show the dependence of $C_U(\lambda)$ on λ for some typical configuration $\{U_{x\mu}\}$. One can see a sharp pronounced peak (cusp) at $\lambda_c \sim 0.8$. Of course, the position and size of this peak depend somewhat on the choice of configuration. However, this peak demonstrates rather weak dependence on the volume (compare 8^4 and 10^4 data).

Let us define the variance $\sigma(F)$ in a standard way

$$\sigma^2(F) = \frac{1}{N_U} \sum_i^{N_U} \tilde{F}_i^2 - \left(\frac{1}{N_U} \sum_i^{N_U} \tilde{F}_i \right)^2. \quad (2)$$

Figure 3 demonstrates the dependence $\sigma(F)$ on λ for 8^4 lattice. For comparatively small values of λ , i.e. till values $\lambda \lesssim 0.6$ where the strong coupling approximation for $\langle F \rangle$ works well, this variance is practically stable. However, for λ 's between 0.65 and 0.8 we can see a drastic increase of the variance.

4 Summary and discussion

To summarize, we have performed an exploratory study of the pure gauge $SU(2)$ theory with the interpolating gauge *à la* Parrinello–Jona-Lasinio–Zwanziger with the gauge fixing functional defined in eq.(3). Therefore, this gauge interpolates between no–gauge case and maximally Abelian gauge.

Our data indicate on the existence of the strong non–analyticity with respect to λ (phase transition) at $\lambda_c \sim 0.8$. Most probably, the mechanism of this transition is similar to that in the spin–glass models. At the moment it is rather difficult to specify the order of this phase transition. It is interesting to note that the existence of a transition with respect to the interpolating parameter λ has been also found for another choice of the functional $F = F_{LG}$ [8] which corresponds to the Lorentz (or Landau) gauge at infinite values of λ .

The existence of this transition makes it clear that there is no smooth interpolation between the no–gauge case and the case with MAG. This observation is of importance for gauge dependent objects (e.g. σ_{abel}) especially taking into account that the Abelian Projection $\{U_{x\mu}\} \rightarrow \{h(U_{x\mu})\}$ is (not very well controllable) approximation. We conclude that the ‘physics’ of Abelian Projection is supposed to be different at $\lambda = 0$ (where $\sigma_{abel} = \sigma_{SU(2)}$) and the case with MAG.

The above conclusion needs further confirmation. Finite volume effects must be better studied as well as the dependence of other observables (e.g. σ_{abel}) on λ . This work is in progress.

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